and the $\Delta T=\frac{1}{2}$ rule say that the final three-pion isotopic spin states must be symmetric, giving amplitudes of 1 and 2 for τ' and τ decays, respectively. Thus the τ'/τ branching ratio is $\frac{1}{4}$. The correction for the difference in phase space in the τ' and τ modes contributes a factor[14] of 1.298, making the τ' branching ratio equal to

$$B(\tau')/B(\tau)=\tfrac{1}{4}(1.298)=0.325\ .$$

The experimental value is 0.350 ± 0.039 which is consistent with the $\Delta T=\frac{1}{2}$ rule.

$K_{\mu3}+K_{e3}$ Branching Ratio

A test of the leptonic $\Delta T=\frac{1}{2}$ rule comes from comparing K_2^0 and K^+ decays.[15,16] The $\Delta T=\frac{1}{2}$ rule predicts

$$\Gamma(K_2^0\to\pi^\pm+e^\mp+\nu)+\Gamma(K_2^0\to\pi^\pm+\mu^\mp+\nu) =2[\Gamma(K^+\to\pi^0+e^++\nu)+\Gamma(K^+\to\pi^0+\mu^++\nu)].$$

Luers *et al.*[15] quote a weighted average of world data up to 1964 for the left side of the equation of $9.9\pm2.0\times10^6$/sec, while Mann[17] quotes a later experimental value of $13.3\pm2.5\times10^6$/sec. A weighted average of these two yields:

$$\Gamma(K_2^0\to\pi^\pm+e^\mp+\nu)+\Gamma(K_2^0\to\pi^\pm+\mu^\mp+\nu) =(11.2\pm1.6)\times10^6/\text{sec}.$$

For the right side of the equation we obtain

$$2[\Gamma(K^+\to\pi^0+e^++\nu)+\Gamma(K^+\to\pi^0+\mu^++\nu) =(12.6\pm1.0)\times10^6/\text{sec}$$

using the K^+ lifetime quoted by Barkas and Rosenfeld.[18]

The difference between these two is $(1.4\pm1.9)\times10^6$/sec, consistent with $\Delta T=\frac{1}{2}$.

[14] R. H. Dalitz, Proc. Phys. Soc. (London) **A69**, 527 (1956).
[15] D. Luers, I. S. Mittra, W. J. Willis, and S. S. Yamamoto, Phys. Rev. **133**, B1276 (1964).
[16] H. H. Bingham, CERN Internal Report CERN/TC/Physics 64-13 (unpublished).
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[18] W. H. Barkas and A. H. Rosenfeld, University of California Radiation Laboratory Report UCRL-8030 Rev., 1963 (unpublished).

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Study of the Three-Body Leptonic Decay Modes of the K^+ Meson*

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K_{e3}^+ and $K_{\mu3}^+$ decay spectra are studied. The results strongly favor vector coupling even if the form factors are allowed arbitrary energy dependence. The relative amplitudes of scalar and tensor couplings are less than 0.3 with the most probable value around 0. The muon and electron couplings are found to be the same within the 14% error quoted. All energy dependences of the form factors are found to be small. If $f_V=A(1+\lambda q^2/M_\pi^2)$ and $g_V=B$, where $q^2=(p^K-p^\pi)^2$, then $\lambda=-0.020\pm0.027$, $B/A=-0.54\pm0.35$.

I. INTRODUCTION

In this paper we examine in detail the three-body leptonic decay modes of the K^+meson

$$(1)\quad K^+\to e^++\nu+\pi^0\quad(K_{e3}^+)\,,$$

$$(2)\quad K^+\to\mu^++\nu+\pi^0\quad(K_{\mu3}^+)\,.$$

In the preceding paper[1] (hereafter called I) we found the branching ratios for the above modes to be 4.7%, and 3.0%, respectively.

Two general but related types of information are obtained in this experiment. The first concerns the nature of the decay interaction, in particular the type of coupling responsible for the decay. Previous experiments[2–4] indicate that the interaction is consistent with pure vector coupling and that pure scalar and pure tensor couplings are much less likely to be present than vector. Further confirmation on this point is available in the current investigation, and limits on the amount of scalar and tensor mixing with the dominant vector coupling are also obtained. The second type of information relates to the energy dependence of the form factors needed to describe the interaction and to the coupling constants involved.

We wish to emphasize that this experiment has been done on a different sample of film from that used for our previous leptonic decay spectrum work,[2] and has

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[1] F. S. Shaklee, G. L. Jensen, B. P. Roe, and Daniel Sinclair, preceding paper, Phys. Rev. **136**, B1423 (1964).
[2] J. L. Brown, J. A. Kadyk, G. H. Trilling, R. T. Van de Walle, B. P. Roe, and D. Sinclair, Phys. Rev. Letters **7**, 423 (1961).
[3] D. Luers, I. S. Mittra, W. J. Willis, and S. S. Yamamoto, Phys. Rev. Letters **7**, 255 (1961).
[4] P. E. Condon, Princeton University Palmer Physical Laboratory and Naval Ordnance Laboratory Technical Report No. 32, (unpublished).

used scanning criteria, bias corrections, etc., which differ sufficiently from our previous work that we feel that this should be considered an independent measurement rather than an extension of the previous work. Further, since we felt that the bias corrections were somewhat better in the present experiment, we have not attempted to statistically combine the two results; we feel our present values to be the best estimates of the parameters measured. It should be further noted, however, that the present results are completely consistent with those obtained previously.

II. THEORETICAL BACKGROUND

Several assumptions are made when attempting to describe the decay interaction, including:

(1) The neutrino wave function v_ν occurs only in the form $(1+\gamma_5)v_\nu$ (two-component neutrino).

(2) One can neglect terms beyond the lowest order in the weak interactions.

(3) Final-state interactions are negligible.

(4) The neutrino and charged lepton are produced at the same vertex.

(5) The interaction equations are invariant under time reversal.

(6) The couplings do not involve derivatives of the lepton fields.

Assumption (5) puts reality conditions on the form factors. A test of assumption (4), the locality assumption, is discussed later. The spin directions of the leptons are not observable in this experiment, so averages over these spins are always taken. It should also be remarked that because of assumption (1), one cannot distinguish between vector and axial-vector couplings or scalar and pseudoscalar couplings. Thus, only vector, scalar and tensor couplings will be considered as possibly being present. A diagram for the interaction is shown in Fig. 1, where the strong interaction part occurs in the box.

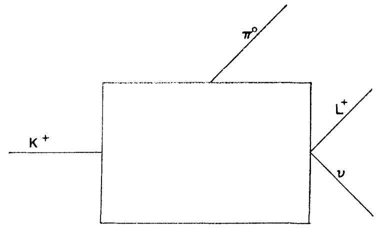

FIG. 1. Diagram for K_{L3}^+ decay.

In the rest system of the K^+ meson there are two independent variables which specify the decay kinematics. These variables may be taken to be P_π, the magnitude of the pion momentum, and θ, the angle between the momentum vectors of the neutrino and pion. The most general form of the joint distribution function for P_π and θ, in the context of the assumptions listed above, may be written as follows[5]:

$$
\begin{aligned}
F(P_\pi,\theta)dP_\pi d\cos\theta
&=\frac{P_\pi^2(W^2-P_\pi^2-M_L^2)^2}{E_\pi(W+P_\pi\cos\theta)^4} f_S'^2(W+P_\pi\cos\theta)^2\\
&\quad+f_V^2(P_\pi^2\sin^2\theta+M_L^2)+f_T^2(P_\pi^2/M_K^2)\\
&\quad\times(P_\pi+W\cos\theta)^2+M_L^2\sin^2\theta\\
&\quad-(f_S'^*f^V+f_S'f_V^*)M_L(W+P_\pi\cos\theta)dP_\pi d\cos\theta,
\end{aligned}
\qquad (1)
$$

where M_K = mass of K^+ meson, M_L = mass of charged lepton, P_π = magnitude of pion momentum, E_π = energy of pion, $W=M_K-E_\pi$, θ = angle between pion and neutrino directions, $f_S'=f_S+(M_L/M_K)g_V$; f_S, f_T, f_V, g_V = scalar functions called "form factors,"[6] which depend only on q^2, the square of the four-momentum transfer (or equivalently, on the energy of the pion). There are two such functions for the vector case. Two other cross terms in Eq. (1) vanish because the assumption of time reversal invariance requires f_S and f_V to be relatively real, whereas f_S and f_T must be relatively imaginary.[5] Comparison of the experimental distributions of P_π and $\cos\theta$, or quantities directly related to them, with the predictions of Eq. (1) will be made to determine the couplings involved and the energy dependences of the form factors. In the analysis, f_S and f_V are both taken to be real.

III. EXPERIMENTAL DETAILS

The data for this experiment came from bubble-chamber film which, along with the scanning and measuring methods, has been described in I. In addition to the film used there, however, 13 more rolls (200 frames per roll) were scanned for the present experiment. The scanning of these 13 rolls was done by a physicist, and the scanning rules were the same as described in I, except that only events which could be K_{e3}^+ or $K_{\mu3}^+$ events were selected for measurement. After measurement, the events were all rescanned, the more difficult ones by two physicists.

The selection criteria were somewhat different than those described in I and are given below. They include no dip angle cutoffs, and frames containing up to six K^+ decays are used.

K_{e3}^+ *sample.* The final sample of K_{e3}^+ decays is composed of 407 events which satisfy the following criteria:

(a) The decay point lies in the fiducial volume defined by the region at least 5.0 cm from any chamber wall or window.

(b) The parent K^+ meson decays at rest as evidenced by the density and scattering of the last few centimeters of K^+ meson track.

(c) The charged secondary is unambiguously identi-

[5] A. Pais and S. B. Treiman, Phys. Rev. **105**, 1616 (1957).

[6] There unfortunately are several standard notations for the two vector form factors. Some others which are common are ($f_+=f_V$ and $f_-=2g_V+f_V$) or ($f_1=f_V+g_V$ and $f_2=-g_V$).

fied as an electron by virtue of either bremsstrahlung or typical electron scattering.

(d) Two electron pairs point unambiguously to the decay point.

(e) Both electron pairs have conversion distances of at least 0.5 cm and potential paths in the chamber after conversion of at least 2.0 cm.

$K_{\mu3}^+$ sample. The final sample of $K_{\mu3}^+$ decays contains 141 events which satisfy the following criteria:

(a) The K^+ meson decay occurs at rest at least 5.0 cm from any wall or window of the chamber.

(b) Two electron pairs point unambiguously to the decay point.

(c) Both electron pairs have conversion distances of at least 0.3 cm and potential paths in the chamber of at least 2.0 cm.

(d) The charged secondary unambiguously stops and decays in the chamber.

(e) The charged secondary neither interacts inelastically nor decays via an obvious $\pi^+-\mu^+-e^+$ chain.

(f) The charged secondary has a range of at least 1.0 cm (this is done mainly to avoid confusion with K_{e3}^+ events, but also because short range events are difficult to measure accurately).

(g) The decay kinematics are not consistent with being $K_{\pi2}^+$ kinematics (this consideration does not include range).

(h) The decay can be reconstructed as a $K_{\mu3}^+$ event.

(i) Events whose charged secondaries have measured ranges of less than 8.5 cm and which, therefore, could be τ' events, are required to have $|\cos\alpha|<0.7$, where α is the angle between the pion and either of the gamma-ray directions as measured in the pion center-of-mass system (the motivation for this requirement is discussed later).

IV. MEASUREMENTS AND BIASES

The measured quantities were the directions of the two gamma rays from the π^0 decays (as determined from the two electron pairs and the K^+ decay point), the direction of the charged secondary, and, for the $K_{\mu3}^+$ events, the range of μ.

For the K_{e3}^+ decays, the events were not kinematically determined, but information was obtainable from ϕ, the angle between the gamma rays, and θ', the angle between the electron and the line bisecting the gamma rays. The first of these is highly correlated statistically with the π^0 momentum and the latter with the electron-π^0 angle. For $K_{\mu3}^+$ decays, the events were determined kinematically with a double-valued solution (zero constraints).

First consider biases common to both $K_{\mu3}^+$ and K_{e3}^+ events. About 7% of the K^+ decays occur in flight. Usually, however, it is possible to tell from the ionization of the primaries that such events did not occur at rest. It is estimated that less than 2% of the final K_{e3}^+ sample comes from decays in flight. For the $K_{\mu3}^+$ sample the figure is even lower, as $K_{\mu3}^+$ decays in flight usually cannot be reconstructed as possible decays at rest. No corrections were made for the inclusion of any such decays.

The average probability for both gamma rays to convert in the chamber varies about 5% over the range of π^0 energies available here and is approximately linear in the π^0 energy. This bias was corrected for in our spectra as was the smaller bias against high-energy pions caused by the minimum conversion distances chosen for the electron pairs. This bias was also approximately linear in the π^0 energy. By comparison with the known spectrum of gamma rays for $K_{\pi2}^+$ decays when gamma conversion probabilities were taken into account, it was determined that for this sample of events no appreciable scanning bias against low-energy gammas was present. This latter result differs from I because all events considered here were scanned and checked by two physicist whereas they were not in I.

Fifteen highly dipped events were rejected because an unambiguous interpretation between the K_{e3}^+ and $K_{\mu3}^+$ modes could not be made.

Distributions of spatial errors in measuring the coordinates of gamma ray vertices relative to the K^+ decay points were obtained from a sample of points measured three times and are described by standard deviations $\sigma_y=\sigma_x=0.029$ cm and $\sigma_z=0.061$ cm.

Next consider biases peculiar to K_{e3}^+ decays. The distribution of ϕ contained a small bias in the large-angle portion because of the lower conversion probability of low-energy gammas. To avoid the effect of this, the ϕ distribution was cut off at 140°. This value was chosen by fitting theoretical parameters for several cutoff values and finding where the parameter stopped varying as a function of cutoff angle. Only seven events were discarded by this process.

A bias of less than three events was expected from $K_{\mu3}^+$ events with very short μ range erroneously called K_{e3}^+. This was considered negligible.

The electron detection efficiency for this part of the experiment was not quite as high as in I but was determined to be >0.95. This bias was ignored since as pointed out previously[2] the measured quantities should largely be independent of detection efficiency.

Monte Carlo error distributions in ϕ as a function of ϕ and conversion distance were obtained as well as the distribution of angular error in the measured direction of the electron secondary as a function of the segment of track used for the direction determination.

Finally consider biases in the $K_{\mu3}^+$ modes. An important bias but one taken into account quantitatively comes from the decay $K^+\rightarrow\pi^++\pi^0+\pi^0(\tau')$ in which only two of the four gamma rays from the π^0 mesons convert into electron pairs within the chamber. Eighty-six such events are expected among the candidates for the $K_{\mu3}^+$ sample. Some of these would fail $K_{\mu3}^+$ kinematics, and most of those that pass would have

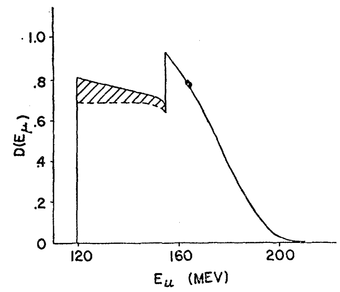

FIG. 2. The function $D(E_\mu)$. The shaded area is the result of τ' contamination.

$|\cos\alpha|>0.7$, where α is the decay angle of the gamma rays in the π^0 center-of-mass frame.[2] There are two reasons for the latter effect. The two electron pairs often come from different π^0 mesons, and hence the probability for ϕ to be 0° or 180° is much greater than for $K_{\mu3}^+$ events. Also, in the case where both electron pairs do come from the same π^0, the calculated π^0 momentum (assuming the event to be a $K_{\mu3}^+$ decay) is usually much greater than the actual π^0 momentum, and the calculated value of α would thus tend to fall near 0° or 180°. The maximum range of a secondary from a τ' decay is slightly under 8.5 cm. Hence a cutoff was introduced, and all $K_{\mu3}^+$ candidates with charged secondary range of less than 8.5 cm and $|\cos\alpha|>0.7$ were discarded. Seven τ' events are still expected to appear in the final $K_{\mu3}^+$ sample; a correction based on a Monte Carlo calculation is made for these. It should be noted here that the chance of identifying a stopping π^+ meson, i.e., seeing the π-μ-e chain, is small in the xenon chamber as the range of the μ is only 0.12 cm.

The $K_{\pi2}^+$ mode, with a large branching ratio, can also simulate $K_{\mu3}^+$ events if the π^+ undergoes an inelastic scattering and stops in the chamber. All $K_{\mu3}^+$ candidates consistent with $K_{\pi2}^+$ kinematics were therefore rejected, and the bias thus introduced was corrected for. This bias is significant in the π^0 energy spectrum but is almost nonexistent in the muon spectrum. About 5% of all $K_{\mu3}^+$ events with stopping secondaries are expected to pass the $K_{\pi2}^+$ kinematics test. It is estimated that less than two $K_{\pi2}^+$ decays remain in the final $K_{\mu3}^+$ sample.

A function $D(E_\mu)$ (see Fig. 2) is defined as a multiplicative factor in the joint distribution of E_μ and P_π to take into account effects, geometric and otherwise, which directly affect the muon spectrum. The major effect is a geometrical cutoff on muon range which was determined by using a set of 258 representative decay points to determine the potential path distribution. Events with muon range of less than one centimeter were discarded both to avoid contamination from false events and because they are hard to measure accurately. The suppression of $D(E_\mu)$ for $E_\mu<152$ MeV is due to the rejection of events with μ range <8.5 cm and $|\cos\alpha|>0.7$. $D(E_\mu)$ also includes the correction for the residual τ' contamination and the bias due to the fact that decays with long secondaries tend to occur toward the edge of the chamber and hence are less likely to have both gamma rays convert.

Some real events fail $K_{\mu3}^+$ decay reconstruction due to measurement error even when small perturbations of their coordinates are made. Monte Carlo calculations, utilizing known measurement error distributions, showed that this occurred more often for events with low π^0 momentum, and a small (about 2%) correction for this was made.

Since the $K_{\mu3}^+$ reconstructions are double valued, it was necessary to attempt to choose between solutions by making crude estimates of the relative gamma-ray energies from the size of the gamma-ray showers. The events were then put in 3 categories: (1) unambiguous (66%); (2) definitely favor one solution but difficult to be certain (16.3%); (3) ambiguous (17.7%). Very frequently, the two solutions for the ambiguous events were practically equivalent. Both solutions for the events in the ambiguous set were retained with weight one-half, while only the selected solutions, with weight unity, were retained for the unambiguous events. Combining set 2 with first the unambiguous and then the ambiguous set resulted in practically no change in the distribution of π^0 momentum, indicating that the choice of solutions was not critical for this subset. The events in set 2 were thus retained in the unambiguous set for the analysis. It was felt that no systematic bias was generated by the above indicated choice of solutions.

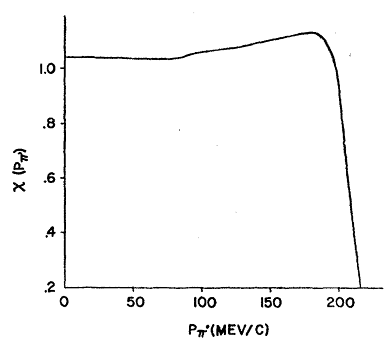

FIG. 3. The function (P_π), which represents the combined relative biases in P_π.

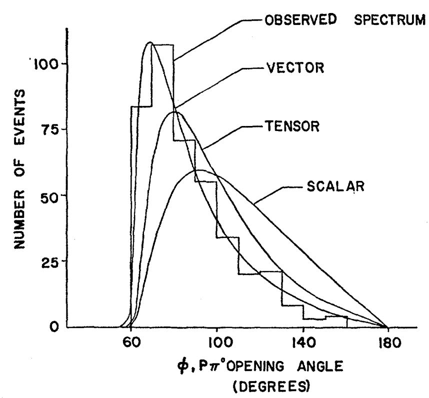

FIG. 4. Distribution of ϕ for V, T, S, couplings with constant form factors, compared with experimental results.

A function χ (P_π) is defined which embraces all the combined relative biases in pion momentum described in the preceding paragraphs. It is included as a multiplicative factor in the joint E_μ, P_π distribution along with $D(E_\mu)$. It is seen plotted in Fig. 3. The major contributions to $\chi(P_\pi)$ come from the τ' bias and the rejection of events that passed $K_{\pi 2}{}^+$ kinematics.

Estimates of the errors in π^0 momentum (typically a few MeV) were made by a Monte Carlo technique using known spatial-error distributions. The errors depended essentially only on the pion momentum itself and the smallest of the two gamma-ray conversion distances.

V. RESULTS

Tests for Pure Couplings

In this section, we assume that only pure couplings may be present. We begin with the $K_{e3}{}^+$ decays and initially assume the form factors to be constant. In $K_{e3}{}^+$ decays, the g_V term is multiplied by M_e/M_k and will be ignored. Figure 4 shows the distributions of ϕ for the various predictions compared to our results. The pure vector case is the only one that fits reasonably well (χ^2 probability of 64%). However, it is still possible that one of the others, say scalar, is correct, and that the lack of fit is due to an energy dependence of the form factor f_S.

The result of a test which is independent of the form-factor energy dependence is shown in Fig. 5. $K_{e3}{}^+$ events are generated by Monte Carlo methods which require the distribution of ϕ to agree with the experimental spectrum. This is done by choosing the pion momentum according to vector coupling with constant form factors and is equivalent to assigning the correct energy dependence to the form factors (to within the accuracy of the data). The distribution of θ' is then uniquely determined by the particular type of coupling. This procedure results in a χ^2 fit of 50% for pure vector coupling but less than 0.1% for both scalar and tensor. Limits on the magnitude of the scalar and tensor amplitudes relative to the vector amplitude are set later on in the analysis; otherwise pure vector is assumed to be the correct coupling in $K_{e3}{}^+$ decays.

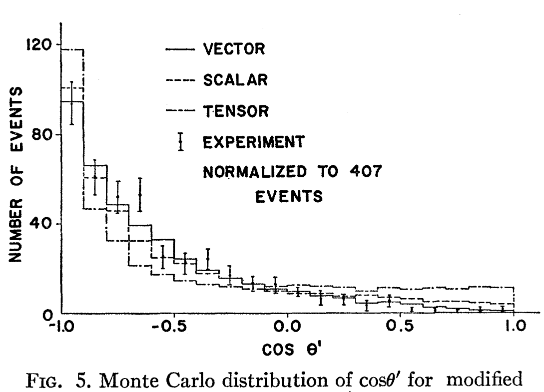

FIG. 5. Monte Carlo distribution of cosθ' for modified V, T, and S couplings. $K_{e3}{}^+$ decays.

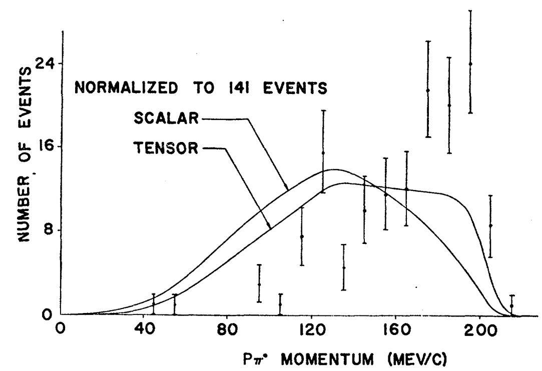

FIG. 6. Predictions of pion momentum distributions, $K_{\mu 3}{}^+$ decay, for tensor and scalar couplings. Form factors are constant. Experimental data are also shown.

Next we consider tests for pure coupling in $K_{\mu 3}{}^+$ decays. Figures 6 and 7 show representative distributions for P_π, the pion momentum, for constant form factors. It is seen that tensor and scalar give poor fits (χ^2 probability less than 0.1%) while pure vector coupling gives a good fit for g_V/f_V in the neighborhood of -1. Again it may be that scalar or tensor is still correct, the poor fit arising because the form factor is not constant.

A test which circumvents the possible energy dependences can be made by a procedure analogous to that used for the $K_{e3}{}^+$ decays. The resulting predicted distributions of muon energy may then be compared with experiment to give an absolute test of the coupling (see Fig. 8). The result gives a χ^2 probability of 73% for vector, 4.5% for tensor and 40% for scalar. Although pure scalar coupling is not excluded, the energy dependence needed for it is much larger than any estimates of form-factor energy dependence seem able to give.

For the remainder of the analysis, it will be assumed that pure vector coupling is responsible for the $K_{\mu 3}{}^+$ decay interaction. The $K_{\mu 3}{}^+$ data are insufficient to provide meaningful tests of the amount of scalar and tensor mixing present.

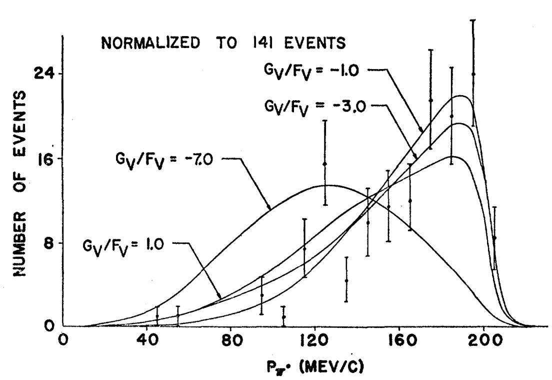

FIG. 7. Predicted $P_\pi{}^0$ spectra in xenon bubble chamber for vector coupling, constant form factors for various values of g_V/f_V, compared to observed spectrum ($K_{\mu 3}{}^+$ decays).

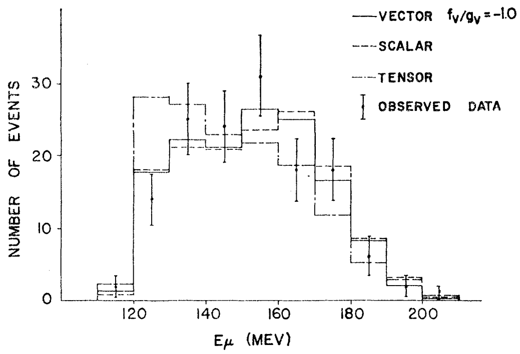

FIG. 8. Monte Carlo distributions for energy of muons from $K_{\mu3}^+$ decays, for modified V, T, S couplings and constant form factors. Corrected for τ' contamination.

Form-Factor Analysis in K_{e3}^+

In general, under the basic assumptions given in Sec. II, the form factors in K_{e3}^+ (or $K_{\mu3}^+$) decay are arbitrary functions of q^2, where q^2 is the square of the four-momentum transfer in the decay, i.e.,

$$q^2=(P^K-P^\pi)^2,$$

and where P^K and P^π are the four-momenta of the K^+ and π^0, respectively. For K^+ decays at rest, one gets

$$q^2=M_K^2+M_\pi^2-2M_KE_\pi.$$

If the dependence on q^2 is assumed small, as several current theoretical models predict[7,8] and as the π^0 opening angle spectrum verifies, one may express f_V^e (the only form factor of interest in K_{e3}^+ decay) in a limited series expansion as follows:

$$f_V^e=A_e(1+\lambda_e(q^2/M_\pi^2)). \tag{2}$$

Here A_e is a constant and λ_e is a unitless parameter which must be small if the expansion (2) is to be valid. The value of q^2/M_π^2 varies from nearly 0.0 to 7.066 in K_{e3}^+ decay, so if λ_e should exceed about 0.1, then the above expansion would not be useful. The goal of the analysis is to determine the best value for λ_e.

The method selected is to find the value of λ_e which gives the best fit to the π^0 opening angle (ϕ) distribution. The best value obtained in this manner is $\lambda_e=-0.010\pm0.029$, which is consistent with the value we previously obtained,[2] which was $\lambda_e=0.038\pm0.045$.

Form Factor Analysis in $K_{\mu3}^+$ Decay

In analogy with the expansion of the form factor in K_{e3}^+ decay above, the form factors in $K_{\mu3}^+$ decay are taken of the form

$$f_V=A_\mu(1+\lambda_\mu(q^2/M_\pi^2)), \tag{3a}$$

$$g_V=B_\mu(1+\lambda_\mu'(q^2/M_\pi^2)). \tag{3b}$$

[7] P. Dennery and H. Primakoff, Phys. Rev. **131**, 1334 (1963).
[8] J. D. Jackson Lectures given at the Physics Division, Argonne National Laboratory, 1961 (unpublished).

This expansion seems justified, since the observed P_π and E_μ distributions are consistent with constant form factors for certain values of g_V/f_V, and since the expansion worked very well in the case of K_{e3}^+ decays.

We at first attempted to determine the values B_μ/A_μ, λ_μ, λ_μ' in a manner independent of the $K_{\mu3}^+$ decay rate, this time using a two-dimensional likelihood function. The results, however, were not sensitive to the value for λ_μ', so λ_μ' was set to zero and kept there for the rest of the calculations. The likelihood function is then found to be a maximum at the following values:

$$B_\mu/A_\mu=-0.20\pm1.0,$$
$$\lambda_\mu=-0.052\pm0.07.$$

The uncertainties quoted above are very closely correlated; i.e., the combinations of values for λ_μ and B_μ/A_μ which give good agreement to the experimental spectra cover a wide range. The uncertainties are estimated by observing the behavior of the logarithm of the likelihood function along the direction in which it changes most slowly.

The hypothesis of μ-e universality states that, not only are the μ and e interactions of the same form, but the couplings in these interactions are also of the same strength for both particles. In particular, μ-e universality would predict that $A_e=A_\mu$ and $\lambda_e=\lambda_\mu$. To test this hypothesis, an appropriate likelihood function is defined which utilizes the experimental result, obtained in I, that

$$R(K_{\mu3}^+)/R(K_{e3}^+)=0.63\pm0.10.$$

The likelihood function is a maximum for $A_\mu/A_e=1.08\pm0.14$, and at this point the values for λ_e, λ_μ are $\lambda_e=-0.01\pm0.03$; $\lambda_\mu=-0.05\pm0.065$. These values are all consistent with universality.

It is now assumed from the above results that A_μ/A_e is unity and that $\lambda_e=\lambda_\mu$, as predicted by μ-e universality. After using many combinations of B_μ/A_μ and $\lambda_e=\lambda_\mu$ to determine where the logarithm of the likelihood function is a maximum for this situation, the following results are obtained:

$$B_\mu/A_\mu=-0.54\pm0.35,$$
$$\lambda_e=\lambda_\mu=-0.020\pm0.027.$$

Assuming the branching ratios given in I, we then have

$$A_e=A_\mu=(7.7\pm0.6)\times10^{-2}(\text{MeV})^{-2}\,\text{sec}^{1/2},$$
$$B_\mu=-(4.2\pm2.8)\times10^{-2}(\text{MeV})^{-2}\,\text{sec}^{1/2}.$$

Various ways to give rough predictions of the values of B_μ/A_μ, $\lambda_\mu=\lambda_e$ and λ_μ' have been outlined in the literature. Several of these are discussed in Refs. 7 and 8 and further references are also given there. Dennery and Primakoff[7] give several alternative forms depending on the assumptions made. One of the forms derived (and the only one which will be mentioned here) is based on the assumption that the two known K-π resonances,

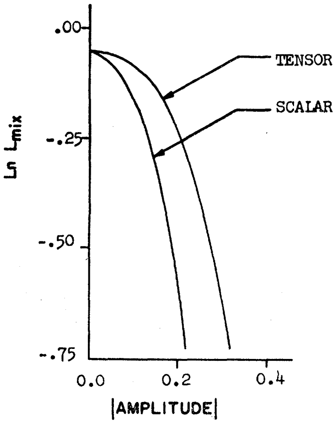

FIG. 9. Ln L_{mix} for constant form factors ($\lambda=0$), as a function of $|S|$, $|T|$.

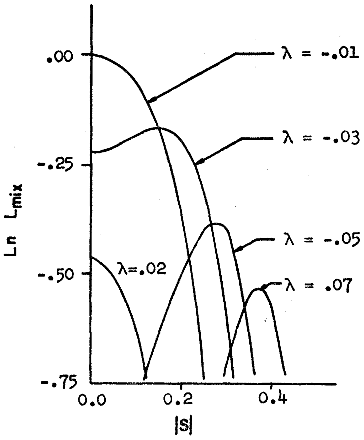

FIG. 10. Ln L_{mix} as a function of $|S|$ for $|T|=0$.

namely, the spin-one, negative-parity states at 730 and 880 MeV, are responsible for the dominant contributions to the form factors. Then they show that this implies (in the current notation)

$$\lambda_\mu(M_K^2/M_\pi^2)\approx-(2B_\mu/A_\mu+1)\,.$$

For $\lambda_\mu=-0.02$, this would predict

$$B_\mu/A_\mu=-0.36\,,$$

which is close to the measured value.

Mixed Coupling in K_{e3}^+ Decay

Next we attempt to set limits on the amount of scalar and tensor coupling that could be present in K_{e3}^+ decay if pure interactions are not assumed. Equation (1) gives the joint distribution function of P_π and $\cos\theta$ for the most general case of arbitrary coupling. Two further assumptions serve to simplify this equation:

(1) Terms containing a factor M_e may be ignored, where M_e is the electron mass; this eliminates the remaining cross terms in Eq. (1).

(2) The form factors all have the same energy dependence, which is taken of the form

$$f_i=(A_i/N_i)(1+\lambda q^2/M_\pi^2)\,;\quad i=V,S,T\,.$$

Here the A_i are amplitudes related by

$$A_V^2+A_S^2+A_T^2=1$$

and the N_i are normalization constants chosen such that the distribution functions multiplied by the A_i^2 are individually normalized distribution functions. The distribution of ϕ (π^0 opening angle) is then readily obtained.

For the case of constant form factors ($\lambda=0$), the results of a best fit to the distribution of ϕ are shown in Fig. 9. The limits set on A_S and A_T are

$$|A_S|<0.2$$
$$|A_T|<0.3;$$

with the most likely values very near zero. The limits are chosen such that at those points $\ln L_{\text{mix}}$, where L_{mix} is the likelihood function for a given coupling mixture, is down by at least 0.5 from the maximum value to occur for $\lambda=0$.

For nonzero λ in the form factors, one would not be surprised to find maxima in $\ln L_{\text{mix}}$ for some combinations of λ, A_S, and A_T which are nearly as great or even greater than the maximum for pure vector coupling, since there are more parameters to vary. The maxima in $\ln L_{\text{mix}}$ do tend to rise in the case of tensor coupling as λ becomes more negative, and the maxima occur at larger values for A_S and A_T. The curves are shown in Figs. 10 and 11.

It is evident that a second constraint, namely the θ'-ϕ angular correlations, is needed to set limits on A_S and A_T when λ is allowed to be nonzero. The results of a two-dimensional likelihood function fit using these correlations do, in fact, allow us to limit both A_S and A_T to be less than 0.3 in absolute value. This is for any energy dependence of the form factors, except that all

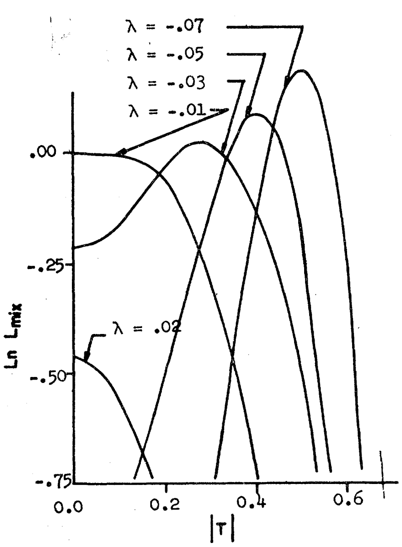

FIG. 11. Ln L_{mix} as a function of $|T|$ for $|S|=0$.

the form factors are restricted to the same form, as described earlier. The predicted angular correlations were necessarily arrived at by Monte Carlo techniques.

ν-e Locality

Finally we may examine the consequences of abandoning the ν-e locality assumption, i.e., allowing the ν and e to be produced at different vertices or allowing final-state interactions (presumably electromagnetic) for them. The result of abandoning this assumption is that the four-vectors P^L and P^ν may appear separately in the coupling and not just in the form P^L+P^ν. At first sight there seem to be two consequences: (1) More form factors can appear as there are now more independent four-vectors. (2) The form factors can depend on another variable, say, $q'^2=(p^K\text{-}p^\nu)^2$, in addition to $q^2=(p^K\text{-}p^\pi)^2$. However, by elementary calculations it can be shown that in the present experiment only one new form factor appears, and we consider only the second possibility. We now make the approximation

$$f_V^2=A^2(1+2\lambda(q^2/M_\pi^2)+2\mu(q'^2/M_\pi^2)),$$

where μ is the parameter indicating a locality violation.

The range of q'^2/M_π^2 is from about 1 up to 13.37, which is roughly double the range of q^2/M_π^2. Thus, for the expansion above to be meaningful, μ must be less than, say, 0.04.

The results of the likelihood calculation made by comparing the experimental results with theoretical distributions of ϕ and θ' obtained by Monte Carlo methods is shown in Fig. 12. The likelihood function here is denoted by L_{lc}.

The curve was sketched in with a French curve. For $\mu<0$, $\ln L_{lc}$ drops off rapidly, so that statistical fluctuations are not a problem. For $\mu>0$, however, $\ln L_{lc}$ does not vary rapidly with μ, and the statistical uncertainties mentioned above seriously cloud the results. To get sufficient statistics to reduce these fluctuations to a more tolerable level would require prohibitive computer time. A peaking does occur near $\mu=0.01$, and with a confidence level of about 90%, one may say that

$$-0.01<\mu<0.03.$$

This can be interpreted as a verification of the validity of the ν-e locality assumption.

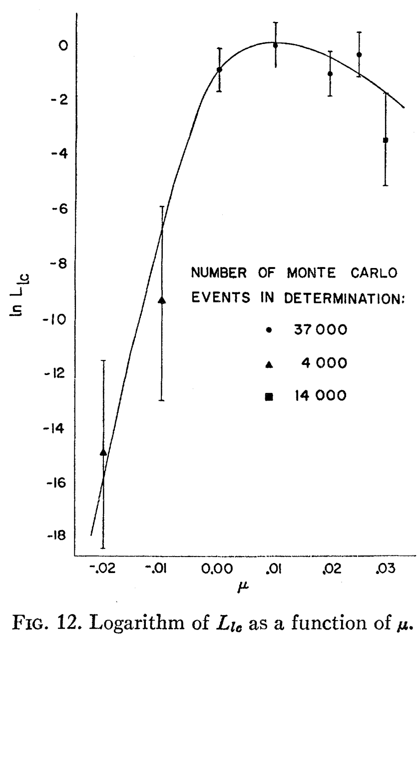

FIG. 12. Logarithm of L_{lc} as a function of μ.

VI. SUMMARY OF CONCLUSIONS

Nature of Coupling

Evidence from the K_{e3}^+ and $K_{\mu3}^+$ decays studied in this experiment strongly supports the hypothesis of pure vector coupling in the decay interaction, even if the form factors are allowed arbitrary energy dependences. The K_{e3}^+ mode is used to set limits of 0.3 on the amplitudes of the scalar and tensor couplings, with the most probable values for these amplitudes near zero.

μ-e Universality

The best value for A_μ/A_e, from the combined K_{e3}^+ and $K_{\mu3}^+$ data and using the observed $K_{\mu3}^+/K_{e3}^+$ relative branching ratio, is

$$A_\mu/A_e=1.08\pm0.14,$$

where A_μ and A_e are proportional to the strengths of the couplings in the two interactions. This is taken as verification of the hypothesis of μ-e universality.

Form Factors

Of the two form factors, f_V and g_V, present in vector coupling, the energy dependence of g_V could not be measured. The energy dependence of f_V is consistent with zero dependence. In particular, if

$$f_V=A(1+\lambda(q^2/m_\pi^2)),$$

then the best over-all value for λ, assuming μ-e universality, is

$$\lambda=-0.020\pm0.027.$$

If we take g_V to be a constant, say B, then the best value for the ratio B/A is

$$B/A=-0.54\pm0.35.$$

In terms of f_+ and f_- this implies that at $q=0$, $f_-/f_+=-0.08\pm0.7$.

Locality Condition

A rough test of the assumption that the neutrino and charged lepton originate at the same vertex indicates the validity of this assumption.